Derivatives
Forwards, Futures, Options and Swaps
Derivatives

- A derivative asset is any asset whose payoff, price or value depends on the payoff, price or value of another asset.
- The underlying or primitive asset may be almost anything – land, stock, interest rates, another derivative asset.

- Derivative are typically priced using “no arbitrage” arguments.
- Arbitrage is a trading strategy that is self-financing (requires no cash) and which has a positive probability of positive profit and zero probability of negative profit.
  - That is, you get something for nothing.
- The price of the derivative must be such that there are no arbitrage opportunities.
Forwards and Futures

- A forward contract is simply a current agreement to a future transaction
  ◦ Payment for and delivery of the asset occur at a future date.

- A forward contract fixes the price of the future transaction
  ◦ a.k.a. the forward price.

- A futures contract is similar, with the following differences:
  1) Futures contracts are standardized for exchange trading.
  2) Gains and losses on futures contracts are recognized daily (marking to market).
Ex. Consider a farmer with a crop of wheat in the ground.
The change in the value of the wheat crop in the ground as a function of the change in the price of wheat is shown in the Figure.

As the price rises (falls) the farmer’s wheat becomes more (less) valuable.
Since the farmer owns the wheat we say he has a long position in the wheat.
Forwards and Futures

- Increase in profit
- Long position in wheat
- Price increase
Forwards and Futures

- Suppose the farmer agrees to sell the wheat under a forward contract at a forward price = the current spot price with delivery upon harvest.
  1. Since the farmer sold the wheat forward, the farmer has a \textit{short} position in the forward contract.

- Future changes in the price of wheat no longer affect the farmer:
  - He has fixed the amount of money he will receive for his crop.

- It is useful however to recognize the changes in value of the crop and the contract separately, even though in this case they offset.

- The separate gains and losses are shown below.
Forwards and Futures

- Increase in profit
- Long position in wheat
- Price increase
- Forward position
Forwards and Futures

- As the price rises, the crop is more valuable
  - But there are offsetting losses in the short position (the farmer sold) in the forward contract.
  - Vice versa when prices fall.
- The net result is profits are constant as prices change.

- Recognizing the separate effects on the value of the crop and contract is relevant in cases where the offsetting effect is not perfect.
  - When there is basis risk
**Forwards and Futures**

- Forward prices have the following relation to spot prices:

  \[ F_t = S_t(1 + c)^T \]

  where:
  - \( F_t \) = forward price
  - \( S_t \) = spot price
  - \( c \) = carrying cost (per period)
  - \( T \) = number of periods

- Carrying costs include:
  - opportunity costs of holding the asset (i.e. what you could have earned investing the proceeds from a current sale)
  - storage costs of holding the asset (if any) less any gains from holding the asset (e.g. benefit of an inventory buffer, yield on a financial instrument) which are called convenience yields.
Forwards and Futures

- The relationship between the spot and forward prices in * is an example of a No Arbitrage condition
  - You can’t make a risk-free profit

- To see this, think about what happens if * does not hold – suppose

\[ F_t > S_t(1 + c)^T \]

- You can sell at \( F_t \), buy on the spot market for \( S_t \) and deliver in \( T \) periods.
- Further, this requires no money – it is self-financing.
- In theory, if * does not hold, you can make an infinitely large profit.

But
  - buying on the spot market drives \( S_t \) up,
  - selling on the futures market drives \( F_t \) down.
Forwards and Futures

- Limitations of Forward Contracts
  - Credit or default risk: Both parties are exposed to the risk that the other party may default on their obligation.
  - Sharing of strategic information: The parties know what specific risk is being hedged.
  - It is hard to determine the market values of negotiated contracts as these contracts are not traded.

- These limitations of forward contracts can be addressed by using exchange-traded contracts such as exchange traded futures, options, and swap contracts.
Futures Contracts
Exchange traded derivatives cannot be customized (like forward contracts) and are available only for specific assets and for limited set of maturities.

A futures contract is a contract to buy or sell a stated commodity (such as wheat) or a financial claim (such as U.S. Treasuries) at a specified price at some future specified time.
These contracts, like forward contracts, can be used to lock-in future prices.

There are two categories of futures contracts:
- **Commodity futures** – are traded on agricultural products, metals, wood products, and fibers.
- **Financial futures** – include, for example, Treasuries, Eurodollars, foreign currencies, and stock indices.

Financial futures dominate the futures market.
Forwards and Futures

- Similar to forward contracts, firms can use futures contract to hedge their price risk.
- If the firm is planning to buy, it can enter into a *long hedge* by purchasing the appropriate futures contract.
- If the firm is planning to sell, it can sell (or short) a futures contract. This is known as a *short hedge*. 
Forwards and Futures

- There are two practical limitations with futures contract:
  - It may not be possible to find a futures contract on the exact asset.
  - The hedging firm may not know the exact date when the hedged asset will be bought or sold.
- The maturity of the futures contract may not match the anticipated risk exposure period of the underlying asset.

- **Basis risk** is the failure of the hedge for any of the above reasons.
  - Basis risk occurs whenever the price of the asset that underlies the futures contract is not perfectly correlated with the price risk the firm is trying to hedge.
  - If a specific asset is not available, the best alternative is to use an asset whose price changes are highly correlated with the asset.
  - For example, hedging corn with soybean future.
  - If the prices of a contract with exact duration is not available, the analysts must select a contract that most nearly matches the maturity of the firm’s risk exposure.
Managing Default Risk

Default is prevented in futures contract in two ways:

◦ **Margin** – Futures exchanges require participants to post collateral called margin.

◦ **Marking to Market** – Daily gains or losses from a firm’s futures contract are transferred to or from its margin account.

### Figure 20.4

**Marking to Market on Futures Contracts**

Upon initiating a futures contract the exchange will require the parties to the contract to post an initial margin. In this example the margin is roughly 16% of the $25,780 (= $2,578 x 10 contracts) value of the 10 contracts. In addition, the exchange requires that the firm not allow the margin account to drop below a maintenance margin level such that the margin account never goes to zero.

<table>
<thead>
<tr>
<th>Date</th>
<th>Margin Account Balance on 10 Contracts</th>
<th>Value of 1 Futures Contract</th>
<th>Change in 1 Futures Price</th>
<th>Change in Value of 10 Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Oct-09</td>
<td>$4,000</td>
<td>$2,578</td>
<td>$(82)</td>
<td>$(820)</td>
</tr>
<tr>
<td>2-Oct-09</td>
<td>3,180</td>
<td>2,496</td>
<td>7</td>
<td>70</td>
</tr>
<tr>
<td>3-Oct-09</td>
<td>3,250</td>
<td>2,503</td>
<td>7</td>
<td>70</td>
</tr>
<tr>
<td>4-Oct-09</td>
<td>3,880</td>
<td>2,566</td>
<td>63</td>
<td>630</td>
</tr>
</tbody>
</table>
Options

- **Terminology**
  An option is a contract in which the *writer* of the option grants to the *buyer* of the option the right to purchase from or sell to the writer a designated asset at a specified price within a specified period of time.
  - writer = seller = short
  - buyer = long
  Option is sold for a price called the **premium**.

- The specified price of the underlying instrument is the exercise or strike price.
  The end of the specified period of time is the expiration date.

- An **American** option can be exercised at any time up to the expiration date.

- A **European** option can be exercised only on the expiration date.
Options

- Terminology

- When the option grants the buyer the right to purchase (sell) the asset it is called a call (put) option.

- The buyer has a right but not an obligation to perform.
  - The writer is obligated to perform if buyer exercises.

- Only option writers must maintain margin.

- The underlying instrument can be virtually anything: stocks, bonds, indexes, currencies, commodities, or futures contracts.
Option Strategies

- **Naked Strategies**

*Buying a call (long call)*

Call option on an asset with an exercise price of 100 and a premium of $3. The profit on the transaction at expiration as a function of the asset price can be depicted:

The payoff at expiration is $\max[S - K, 0]$.
Option Strategies

- *Writing a call (short call)*
- Assuming the same exercise price and premium, the profit on the transaction at expiration as a function of the asset price can be depicted:

\[
\text{The payoff at expiration is}
\]
\[
-\max[S - K, 0] = \min[K - S, 0]
\]
Option Strategies

- *Buying a Put (long put)*
- Put option on an asset with an exercise price of $100 and a premium of $2. The profit on the transaction at expiration as a function of the asset price can be depicted:

\[
\text{Profit} \quad 98 \quad 2
\]

\[
\begin{align*}
98 & \quad 100 \\
S & \quad S
\end{align*}
\]

The payoff at expiration is
\[
\max[K - S, 0]
\]
Option Strategies

- *Writing a put (short put)*
- Assuming the same exercise price and premium, the profit on the transaction at expiration as a function of the asset price can be depicted:

\[
\text{The payoff at expiration is } \quad -\max[K - S, 0] = \min[S - K, 0]
\]
# Option Strategies

<table>
<thead>
<tr>
<th>Naked Strategy</th>
<th>Circumstance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy call</td>
<td>Expect P to rise (r to fall)</td>
</tr>
<tr>
<td>Write put</td>
<td>Expect P won’t (r won’t rise)</td>
</tr>
<tr>
<td>Write call</td>
<td>Expect P won’t rise (r won’t fall)</td>
</tr>
<tr>
<td>Buy put</td>
<td>Expect P to fall (r to rise)</td>
</tr>
</tbody>
</table>
Covered Positions

- *Covered Call Writing*
  - Long in asset
  - Short in call (write call)
Covered Positions

- **Protective Put Strategy**
  - Long in asset
  - Long in put
散发 - 结合长和短的同一类型的期权位置。

例子："多头垂直期权差价"
- 买入低行权价的看涨期权
- 卖出高行权价的看涨期权

Covered Positions

- Spread – combine long and short positions in the same type of option.
- Example: “Bullish Vertical Spread”
  - Buy call with low strike price
  - Sell call with high strike price
Covered Positions

- Combination – combine long or short positions on different types of options.
- Example: Bottom straddle
  - Buy call and put with the same strike price
Covered Positions

- Collar
  - Long in the asset
  - Buy an out-of-the-money put
  - Sell an out-of-the-money call
Covered Positions

- Collar is equivalent to
  - Covered call +
  - Protective put
Options are redundant assets.
- The cash flows from an option duplicate the cash flows from a portfolio of the underlying asset and bonds.
- This implies that the value of the portfolio and the value of the option must be the same.
  - If not, then there is an arbitrage opportunity (free money!)

Cash Flow at Expiration from Buying a Call

<table>
<thead>
<tr>
<th></th>
<th>$S &lt; K$</th>
<th>$S = K$</th>
<th>$S &gt; K$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Call</strong></td>
<td>0</td>
<td>0</td>
<td>$S - K$</td>
</tr>
</tbody>
</table>
Equivalent Positions and Put–Call Parity

- Cash Flow at Expiration from
  - Buying a Put,
  - Buying the Asset and
  - Borrowing the P.V. of the Exercise Price

<table>
<thead>
<tr>
<th></th>
<th>$S &lt; K$</th>
<th>$S = K$</th>
<th>$S &gt; K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put</td>
<td>$K - S$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Asset</td>
<td>$S$</td>
<td>$S$</td>
<td>$S$</td>
</tr>
<tr>
<td>Loan</td>
<td>$- K$</td>
<td>$- K$</td>
<td>$- K$</td>
</tr>
<tr>
<td>Portfolio</td>
<td>0</td>
<td>0</td>
<td>$S - K$</td>
</tr>
</tbody>
</table>
Equivalent Positions and Put–Call Parity

- These two positions have the same payoff in each state of the world and, in the absence of arbitrage, must sell for the same price.

- Thus, we have Put–Call Parity:

\[ C = P + S - K/(1+r) \]

- We can rewrite this as

\[ P = C - S + K/(1+r) \]

- In this form, it is known as the synthetic put relationship.
Equivalent Positions and Put–Call Parity

- This relationship holds for European options and American options if the asset pays no interim cash flows.

- If the stock pays dividends, then the put–call parity relationship for European options is

\[
C = P + S - D - \frac{K}{(1 + r)}
\]

- where \( D \) is the PV of dividends received before the option expires.
A Simple Option Pricing Model

- We will show a simple binomial pricing model to demonstrate how options are priced.
  - The trick in valuing the option is to find “something else” that has exactly the same payoffs as the option.

- If we know the price of that “something else,” then it should sell for the same price as the option.
  - (no arbitrage opportunities).
  - If the option were to sell for any different price, then arbitrage would quickly bring the prices together.

- So, being mysterious about the something else, let us proceed with an example.
A Simple Option Pricing Model

- Wharton stock currently sells for 50 but it could rise to 77 or fall to 23.
- Let us find the value of a call option written on Wharton with a striking price of 52 and that can be exercised in one year.
- First, consider the potential payoffs to the option at maturity.

<table>
<thead>
<tr>
<th></th>
<th>Wharton price = 23</th>
<th>Wharton price = 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call option value (exercise price = 52)</td>
<td>0</td>
<td>25</td>
</tr>
</tbody>
</table>
Now consider what the something else might be.
Imagine you were to purchase 0.46296 of a share of Wharton stack and borrow 9.68, which had to be paid back with 10% interest in exactly one year.
This “portfolio” containing the stock and the debt would have the following value at year end.

<table>
<thead>
<tr>
<th></th>
<th>Wharton price = 23</th>
<th>Wharton price = 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.46296 of share of Wharton stock</td>
<td>10.648</td>
<td>35.648</td>
</tr>
<tr>
<td>Repayment of loan of 9.68 plus 10% interest</td>
<td>(10.648)</td>
<td>(10.648)</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>25</td>
</tr>
</tbody>
</table>
Thus, the value of the option must be:

Value of call = 
  hedge ratio* (asset price) – PV of loan repayment

= 0.46296 (price Wharton share) – 9.68 loan
= 0.46296(50) – 9.68 = 13.468

The payoffs to the option can always be replicated by combining
  ◦ a risk–free position (lending or borrowing) with
  ◦ a position in the underlying asset,

This formula enables you to value the option.
The trick to doing this is to find how much of the underlying asset needs to be included in this portfolio.
A Simple Option Pricing Model

- This proportion is known as the hedge ratio or the option delta (δ).
- It is calculated as follows:

  \[ \text{Hedge ratio} = \frac{\text{spread of option payoffs}}{\text{spread of asset payoffs}} = \frac{25 - 0}{77 - 23} \]
Let us now value a put option on the same stock where the exercise price is, say, 50 (at the money).
The payoff at maturity to the holder of the put option is:

<table>
<thead>
<tr>
<th></th>
<th>Wharton price = 23</th>
<th>Wharton price = 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put option value</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td>(exercise price = 50)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A Simple Option Pricing Model

- Now the hedge ratio, or $\delta$, is $(0-27)/(77-23) = -0.5$.

- But the portfolio positions are reversed.
  - Now you will take a short position in 0.5 shares of Wharton stock and lend 35 at 10% interest.

- The terminal value of this portfolio is:

<table>
<thead>
<tr>
<th></th>
<th>Wharton price = 23</th>
<th>Wharton price = 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short 0.5 of share of Wharton stock</td>
<td>(11.5)</td>
<td>(38.5)</td>
</tr>
<tr>
<td>Repayment of loan of 35 plus 10% interest</td>
<td>38.5</td>
<td>38.5</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>0</td>
</tr>
</tbody>
</table>
A Simple Option Pricing Model

- The portfolio has exactly the same payoffs as the put option and therefore must sell for the same price.

- Thus the value of the put option must be:

  \[
  \text{Value of put} = -0.5 \text{ (price Wharton share)} + 35 \text{ loan} \\
  = -0.5(50) + 35 = 10
  \]

- We can extend the problem to two or more periods.
- The option value is then computed by working back from the end of the “tree”.
The same general principles of option pricing can be used where the asset prices do not simply fall out as one of two values but can, at maturity of the option, be spread over a very large number of values.

The insight is the same: one must establish a portfolio that has the exact same payoff structure as the option (i.e., for each possible value of the underlying asset, the payoff to the portfolio is exactly the same as the option).
Black–Scholes Formula

- This key insight is the meat of Black–Scholes option pricing model. The formula is a little opaque.
- The important thing is to note the variables and how they affect value:

  - Black–Scholes Formula:

    \[ C = SN(d_1) - Ke^{-rT} N(d_2) \]

  - where

    \[ d_2 = d_1 - s\sqrt{T} \]
The two terms in the formula for $C$ are essentially the same as those in the binomial formula. Recall that the binomial pricing formula had two terms:
- the first was the hedge ratio times the current asset price and
- the second was the present value of a loan.

The same is true here.
- The first term, $SN(d_1)$ is the stock price times the hedge ratio.
- The second term $Ke^{-rT}N(d_2)$ is really the present value of a loan.

The hedge ratio, $N(d_1)$ is rather complicated to calculate since the stock price is changing constantly over time.

The expression $N(.)$ is the cumulative normal distribution, reflecting that the final value will follow some distribution.
Black–Scholes Formula

- The relevant variables are:
  - the current price of the underlying asset, $S$,
  - the exercise price, $K$,
  - the discount rate, $r$,
  - the time to maturity, $T$, and
  - the volatility of the asset price, $\sigma$. 

Black–Scholes Formula

- Similarly, the Black–Scholes premium for a put is

\[ P = -SN(-d_1) + Ke^{-rT} N(-d_2) \]

- where

\[ d_2 = d_1 - s\sqrt{T} \]

- The can be obtained by direct derivation or using the formula for the call premium and the put–call parity relationship.
Black–Scholes Formula

- **Effect of dividends**
  - Suppose the stock pays dividends at the constant rate $q$.

  - For European options, we can simply replace the stock price by $S(1 - q)^n$ where $n$ is the number of ex-dividend dates prior to expiration.

  - For American options, the ability to exercise prior to expiration makes the problem more complex.

  - If they are exercised early, American calls should be exercised just before the ex-dividend date.
  - This implies that the value of the call depends on the timing of the dividends.
  - As a result, there is no simple formula like to B–S formula.
Swaps

- A swap is an agreement to one or more exchanges of assets or payments at one or more points in time.

- Hence, a swap is effectively a series of forward contracts.

- The two most common types are:
  - **Currency Swaps** in which the parties agree to exchange payments denominated in different currencies at specified fixed or floating exchange rates.
  - **Interest rate swaps** in which, most commonly, a fixed interest rate obligation is exchanged for a variable rate obligation.

- All swaps are over-the-counter so all swaps involve counter-party risk.
Swaps

- Most U.S. financial institutions offer short term deposits and make long term fixed rate loans, which expose them to interest rate risk if rates rise.
- Most European financial institutions offer long term fixed rate deposits and make variable rate loans, which expose them to interest rate risk if rates fall.
- If the U.S. institution exchanges a set of fixed interest payments for a set of floating interest payments from the European institution, the interest rate risk of both institutions is reduced.
Swaps

- US Depositors
  - Short Term Deposit
  - Interest Payments

- US Financial Institution
  - Fixed Intr. Payments
  - Fixed Interest Payments
  - Fixed Rate Loan

- US Borrowers
  - Fixed Interest Payments

- Eur. Depositors
  - Fixed Rate LT Deposit
  - Fixed Interest Payments

- Eur. Financial Institution
  - Floating Rate Loans
  - Floating Interest Payments

- Eur. Borrowers
Swaps

- To see how swaps can reduce risk, consider the following example of an interest rate swap.

- A car manufacturer has an asset that generates a fixed income stream of 5/yr.
- At the current interest rate of 5%, this has a PV of 100.
- If interest rate change, the value of the asset changes.
- The value of the asset is

\[
V(A) = \frac{5}{r} = (5/.05) + \Delta V(A) = 100 - (5/r^2)\Delta r
\]

- where the second and third equalities follow from a Taylor series expansion of \( V(A) = 5/r \).
Swaps

The firm has two kinds of debt:
- Floating rate debt with a face value of 40
- Fixed rate debt with a 5% coupon rate and face value of 40.

The value of the floating rate debt is

\[ V(D_1) = \frac{r40}{r} = 40 \]

The value of the fixed rate debt is

\[ V(D_2) = 2 / \frac{r}{.05} + V(D_2) = 40 - (2/r^2) \cdot r \]

Then the value of the firm’s equity is

<table>
<thead>
<tr>
<th>V(A)</th>
<th>100 – 5/r^2 Dr</th>
</tr>
</thead>
<tbody>
<tr>
<td>-V(D_1)</td>
<td>- 40</td>
</tr>
<tr>
<td>-V(D_2)</td>
<td>- [40 - (2/r^2)] Dr</td>
</tr>
<tr>
<td>V(E)</td>
<td>20 – (3/r^2) Dr</td>
</tr>
</tbody>
</table>
Swaps

- As interest rates rise, $V(D_1)$ does not change while $V(A)$ and $V(D_2)$ both fall.
- The decrease in $V(D_2)$ partially offsets the decrease in $V(A)$ and reduces the interest rate sensitivity of $V(E)$.
- The firm can further reduce the interest rate sensitivity of equity by replacing the floating rate debt with fixed rate debt.
Swaps

- One way to do this would be to retire the floating rate debt and issue new fixed rate debt.
- Then the value of the debt would be
  \[ V(D_2^*) = 80 - \left(\frac{4}{r^2}\right) \Delta r \]
- and the value of the firm’s equity would be:
  \[ V(E^*) = 100 - \left(\frac{5}{r^2}\right) \Delta r - \left[80 - \left(\frac{4}{r^2}\right) \Delta r\right] - R \]
  \[ = 20 - \left(\frac{1}{r^2}\right) \Delta r - R \]
- where \( R \) is the cost of replacing the floating debt with fixed rate debt.
Another, and possibly less expensive, way to accomplish the same thing is to arrange an interest rate swap.

If the floating rate debt is exchanged for fixed rate debt the difference is

\[ V(D_1) - V(D_2) = [40 - (2 / r^2) \Delta r] - 40 = -(2 / r^2) \Delta r \]

The firm should buy a swap, exchanging the floating payment on a notional debt of 40 for the fixed 5% payment on a notional debt of 40.

The cost of the swap is \( C \).

As a result of the swap, the firm has now effectively purchased an asset with a value of \( (2 / r^2) \Delta r \).
Swaps

Then the value of the firm’s equity is
This accomplishes the same thing as the debt replacement.
The swap is a better choice for the firm if $C < R$.

<table>
<thead>
<tr>
<th></th>
<th>$V(A)$</th>
<th>$100 - 5/r^2 \Delta r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-V(D_1)$</td>
<td>$-40$</td>
<td></td>
</tr>
<tr>
<td>$-V(D_2)$</td>
<td>$[40 - (2/r^2)] \Delta r$</td>
<td></td>
</tr>
<tr>
<td>+ swap</td>
<td>$-C + (2/r^2) \Delta r$</td>
<td></td>
</tr>
<tr>
<td>$V(E)$</td>
<td>$20 - (1/r^2) \Delta r - C$</td>
<td></td>
</tr>
</tbody>
</table>
Swaps

- Pricing an Interest Rate Swap

- A “plain vanilla” interest rate swap exchanges a series of fixed interest rate payments for a series of floating interest rate payments, where the interest payments are calculated by applying the rate (fixed or floating) to a notional principal amount.

- This can be viewed as exchanging a fixed rate bond for a floating rate bond with the same principal amount. However, with an interest rate swap the exchange of principal never takes place, so the principal is notional.
Swaps

- Suppose that the swap contract is a three year agreement with semi-annual payments.
- You are going to give your counterparty X a floating rate bond and X is going to give you a fixed rate bond.
- So X is going to have to make fixed interest payments for the next three years.
- The question is “What is the fair swap (fixed) rate?”
Swaps

- It is important to recognize that the price of the floating rate bond is always par (the bond’s principal value).
- Let’s set the par value to $1000
- Also, assume interest rates are as shown in the table

<table>
<thead>
<tr>
<th>Maturity</th>
<th>.5 yrs</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>2.0%</td>
<td>2.5%</td>
<td>3.0%</td>
<td>3.5%</td>
<td>4.0%</td>
<td>4.5%</td>
</tr>
</tbody>
</table>
Swaps

- Denote the swap rate by $R$.
- We need to find that swap rate that makes the payments on the fixed rate bond equal in value to the floating rate bond (par = $1000)

- The present value of the payments on the fixed rate bond are

$$\frac{1000 \times (R/2)}{(1.02)^{1/2}} + \frac{1000 \times (R/2)}{(1.025)^3} + \ldots + \frac{1000 \times (R/2) + 1000}{(1.045)^3} = 1000$$

- This needs to be solved numerically.
- The solution is $R = 4.39\%$

- Note that the swap is worth ZERO at this point.
- The companies have exchanged bond with equal value.
Swaps

- A “plain vanilla” currency swap

- The plain vanilla currency swap involves exchanging principal and fixed interest payments on a loan in one currency for principal and fixed interest payments on a similar loan in another currency.

- Unlike an interest rate swap, the parties to a currency swap will exchange principal amounts at the beginning and end of the swap.
- The two specified principal amounts are set so as to be approximately equal to one another, given the exchange rate at the time the swap is initiated.
Swaps

- In the figure Dozer Co. is an American corporation with significant sales in Germany and Autowagen is a German corporation with significant sales in America.
- Both companies prefer to issue debt in the country where they are domiciled.
- The currency swap permits them to exchange the currencies obtained from their foreign operations to make payments on their domestic debt.
Swaps

- Euros from operations
- Dozer Co.
- Investors in Dozer’s $ denominated bonds
- Dollar payments
- Euro payments
- Dollars from operations
- Autowagen Co.
- Investors in AW’s euro denominated bonds
Swaps

- Exiting a swap agreement

- Sometimes one of the swap parties needs to exit the swap prior to the agreed-upon termination date.
- This is similar to an investor selling an exchange-traded futures or option contract before expiration.
Swaps

- **Buy Out the Counterparty**
  Just like an option or futures contract, a swap has a calculable market value, so one party may terminate the contract by paying the other this market value. However, this is not an automatic feature, so either it must be specified in the swaps contract in advance, or the party who wants out must secure the counterparty's consent.

- **Enter an Offsetting Swap**
  For example, Company C from the currency swap example above could enter into a second swap with Company E, this time receiving dollars and paying euros.
Swaps

- **Sell the Swap to Someone Else**
  Because swaps have calculable value, one party may sell the contract to a third party. As with Strategy 1, this requires the permission of the counterparty.

- **Use a Swaption**
  A swaption is an option on a swap. Purchasing a swaption would allow a party to set up, but not enter into, a potentially offsetting swap at the time they execute the original swap. This would reduce some of the market risks associated with Strategy 2.