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The Principal-Agent Model When Agents Can Make Mistakes

ABSTRACT

This paper examines the principal-agent model when the agent can make a mistake by providing the "wrong" effort level. How the contract is affected depends on (1) whether mistakes can be positive or negative or are always negative and (2) whether the cost of effort depends on intended effort or actual effort.

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Introduction

In May 2001, a Lehman Brothers trader placing a sell order accidently typed 300 million instead of 3 million, leading to a 2.2 percent drop in the FTSE index. In December 2005 a trader at Mizuho Securities placed an order to sell 610,000 shares of J-COM at one yen each instead of one share at 610,000 yen, leading to a 287 point (1.9 percent) drop in the Nikkei.[1] The May 6, 2010 "flash crash" on the New York Stock Exchange saw the share prices of several companies temporarily drop to zero amidst a 1,000 point intra-day swing in the Dow Jones Industrial Average. At the time, this event was attributed to another fat finger error, erroneous posted prices and lack of controls on automated trading algorithms.[2] In August 2010, a fat finger error led to the suspension of trading in five stocks on the London Stock Exchange. None of these events was triggered intentionally, all are the result of mistakes by traders.

Stahel, et. al. (2010) report that there were 107 wrong-site and 25 wrong-patient medical procedures in Colorado between 2002 and mid-year 2008. These figures imply there are approximately 1,200 wrong-site and 280 wrong-patient procedures annually in the U.S. The main causes of the wrong-patient procedures were errors in diagnosis and errors in communication, while the main causes of the wrong-site procedures were errors in judgment and failure to follow protocols. In a study of North Carolina hospitals, Landrigan, et. al. (2010) found that over 20 of every 100 patients were medically harmed during their hospital admission and that the rate at which patients are harmed has not decreased over time. Leape (2009) reports that the U.S. rates for hospital acquired infection and for adverse drug events are higher than the

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[1] Andriotis and Morgan (2010)

[2] A joint study by the SEC and the CFTC (2010) attributes the flash crash to a single large sell order in a market that was already volatile due to concerns about Greece defaulting on its sovereign debt.

error rate for airline luggage handling. No one claims that airlines intend to lose luggage or claims that doctors and other medical personnel intend to infect patients. Most hospital acquired infections can be prevented by relatively simple measures such as hand washing and changing gloves between patients. The failure to do so is not intentional, but is the result of oversight and mistakes. It is easy enough to think of other examples.[3]

Given that people make mistakes, it is reasonable to expect that contracts would take the possibility of mistakes into account. Despite their prevalence in everyday life, mistakes have received relatively little attention in the literature. Diamond (1974) analyzes random variations in care in a model of negligent torts, examining the relationship between the negligence standard and the intended level of care. Demougin and Fluet (2001) examine the tradeoff between incentives and monitoring in a model where a risk neutral but bankruptcy constrained agent can make mistakes. Prendergast (2002) analyzes the role that consumer complaints play in pointing out agent mistakes, showing that complaints can be an effective monitoring device when the consumer's and principal's interests are aligned.

The purpose of this paper is to examine the principal-agent model when the agent can make a mistake by providing the "wrong" effort level. I analyze the standard and widely used model with linear contracts, exponential utility and normally distributed shocks. If mistakes are any deviation of actual effort from intended effort, then mistakes can be either positive or negative. The connotation of mistakes is negative, that they are due to poor reasoning, carelessness or insufficient knowledge. I distinguish between the cases where mistakes may be both positive and negative and the case where mistakes are always negative. I also distinguish between the cases where the agent's cost of effort depends on the agent' intended effort or on the

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[3] Of course, not all mistakes are bad. The invention of the popsicle, microwave oven, and Slinky were all the result of mistakes (Newsweek, 2010, Thinkquest, n.d.).

agent's actual effort. The cost of effort might depend on intended effort if effort consists of relatively durable actions such as purchasing and installing equipment or developing systems and procedures for operations. Mistakes would then be deviations in the equipment's or system's performance. The cost of effort might depend on actual effort if effort requires continued performance by the agent. If the cost of effort depends on actual effort then variations in the cost of effort creates a natural hedge against variations in the agent's compensation due to mistakes. The presence or absence of this hedge affects the contract offered to the agent.

The next section reviews the principal agent model and establishes the notation. Section 3 analyzes the case where mistake can be positive or negative. Section 4 examines the case where mistakes are always negative. The last section provides brief concluding remarks.

2. The Principal-Agent Model

As the framework for the analysis I employ the widely used model with linear contracts, exponential utility and normally distributed errors (e.g., Bolton and Dewatripont, 2005). The principal's output or gross profit is assumed to be equal to the agent's effort plus an exogenous random shock, $\widetilde{q} = a + \widetilde{\varepsilon}$. The shock is normally distributed with mean zero and variance σ^2_ε. The agent is assumed to be risk averse with constant absolute risk aversion preferences, $u(w, a) = -\exp\{-\rho[w - \gamma(a)]\}$, where ρ is the coefficient of absolute risk aversion, w is the agent's monetary wealth and $\gamma(a)$ is the cost of effort. For simplicity, the cost of effort is quadratic, $\gamma(a) = \frac{1}{2}ca^2$.

The principal is assumed to be risk neutral. The principal's problem is to maximize expected profit net of the cost of the agent's compensation, $E\{q - w\}$, subject to the incentive compatibility constraint

$$a = \text{argmax}_{a'}\, E\{-\exp\{-\rho[w - \gamma(a')]\}$$

and the participation constraint

$$E\{-\exp\{-\rho[w - \gamma(a)]\} \geq u(\overline{w}),$$

where $u(\overline{w})$ is the agent's reservation utility level and $\overline{w}$ is the minimum acceptable certainty equivalent compensation. Contracts are restricted to be linear in performance, $w = f + s\widetilde{q}$, where f is the salary or fixed compensation and s is the agent's share of output.

For the agent, maximizing expected utility is equivalent to maximizing certainty equivalent compensation. Upon substituting into the agent's utility function and taking the expectation, the agent's certainty equivalent compensation is

$$w(a) = f + sa - \tfrac{1}{2}ca^2 - \tfrac{1}{2}\rho s^2 \sigma^2_{\varepsilon}.$$

Then the agent chooses effort is $a = s/c$.

Taking the agent's effort into account, the principal's profit is

(2.1) $\qquad s/c - (f + s^2/c).$

The agent's participation constraint becomes

(2.2) $\qquad f + \tfrac{1}{2}s^2/c - \tfrac{1}{2}\rho s^2 \sigma^2_{\varepsilon} = \overline{w}$

Solving for the optimal linear contract, the agent's share of output is:

(2.3) $\qquad s_0 = 1/[1 + \rho c \sigma^2_{\varepsilon}].$

The salary component of the contract, f_0, is then chosen so the participation constraint is binding. The agent's share of output decreases with the agent's risk aversion, the cost of effort and the variability of performance. Increases in these parameters lead to lower agent effort, lower expected output, lower expected compensation for the agent and lower expected net profit for the principal. All of this is well known.

3. Mistakes Can Be Positive or Negative

A. Costly mistakes. The standard model assumes that the agent always provides the intended level of effort. I assume instead that the agent can make mistakes by providing an actual effort that differs from the intended level of effort; both positive and negative deviations are mistakes.[4] Actual effort is equal to intended effort plus a random mistake, $\tilde{a} = a + \tilde{\upsilon}$. Mistakes are assumed to be normally distributed with mean zero and variance σ^2_υ, and independent of the exogenous shock. Output is then $\tilde{q} = a + \tilde{\upsilon} + \tilde{\varepsilon}$. I assume that mistakes are costly in the sense that the agent's cost of effort depends on the actual, rather than the intended, level of effort, $\gamma(\tilde{a}) = \frac{1}{2}c\tilde{a}^2$.

Substituting into the agent's utility function and taking expectations over both exogenous shocks and mistakes, the agent's certainty equivalent compensation is

$$(3.1) \qquad w(a) = f + sa - \tfrac{1}{2}ca^2 - \tfrac{1}{2}\rho s^2 \sigma^2_\varepsilon - \tfrac{1}{2}\rho\sigma^2_\upsilon (s - ca)^2.$$

Given the contact, the agent chooses the intended effort level $a = s/c$, which is the same as when the agent makes no mistakes. Substituting the agent's intended effort into certainty equivalent compensation, the last term vanishes. The principal's expected profit is (2.1) and the agent's participation constraint is (2.2). The agent's share of output under the optimal linear contract is $s_0 = 1/[1 + \rho c \sigma^2_\varepsilon]$, the same as when the agent never makes mistakes. Mistakes increase the variance of output, but when mistakes affect the cost of effort the agent is able to perfectly hedge the risk.

B. Costless Mistakes. I now assume that mistakes are costless in the sense that the agent's cost of effort depends on the intended level of effort, $\gamma(a) = \frac{1}{2}ca^2$.

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[4] Mistakes are distinct from background risk, which would affect the agent's exogenous outside income. Unlike mistakes, background risk does not directly interact with the terms of the contract, but may affect the terms of the contract through its effect on the agent's risk aversion. See Ligon and Thistle (2008a, 2013) for analyses of background risk in principal-agent models.

Substituting into the agent's utility function and taking expectations over both exogenous shocks and mistakes, the agent's certainty equivalent compensation is

$$(3.2) \qquad w(a) = f + sa - \tfrac{1}{2}ca^2 - \tfrac{1}{2}\rho s^2(\sigma^2_\varepsilon + \sigma^2_\upsilon).$$

Given the contact, the agent again chooses the intended effort level $a = s/c$. Substituting the agent's intended effort into expected profit and the participation constraint, the optimal linear contract gives the agent the share

$$(3.3) \qquad s = 1/[1 + \rho c(\sigma^2_\varepsilon + \sigma^2_\upsilon)].$$

This result is intuitive - the agent's mistakes simply increase the variation of performance and, since the agent cannot hedge the risk, it is optimal to reduce the agent's share of output.

Principals monitor agents to improve the quality of information about the agent's effort. In the linear-exponential-normal model, monitoring is valuable to the principle if it reduces the variance of the exogenous shock. In this model, monitoring leads to stronger incentives, since the agent's share of output is decreasing in σ^2_ε. "Training" is expenditures by the principal to reduce the variance of mistakes, σ^2_υ. When mistakes do not affect the agent's cost of effort, training is valuable to the principal. By decreasing σ^2_υ, training reduces the variability of output and therefore reduces the risk born by the agent. As with monitoring, this allows the principal to provide stronger incentives. When mistakes do affect the agent's cost of effort, training is not valuable to the principal, since the natural hedge means the agent is not exposed to additional risk.

4. Mistakes Are Always Negative

A. Costly Mistakes. Now I assume that the agent makes mistakes by providing an actual effort that is less than the intended level of effort; mistakes are always negative deviations. Actual

effort remains equal to intended effort plus a random mistake, $\tilde{a} = a + \tilde{\upsilon}$. Since mistakes are negative deviations from intended effort, the distribution of mistakes is normal with parameters $\mu = 0$ and σ^2_υ, right-truncated at zero.[5] Let $N(z; \mu, \sigma^2)$ denote the ordinate at z of a normal distribution with mean μ and variance σ^2. Mistakes are independent of the exogenous shock. Output is still $\tilde{q} = a + \tilde{\upsilon} + \tilde{\varepsilon}$, but expected output is reduced to $a - \sigma^2_\upsilon\sqrt{(2/\pi)}$ as a result of mistakes. The contract is linear in output, $w = f + s\tilde{q}$.

I assume the cost of effort depends on actual effort, $\gamma(\tilde{a}) = \frac{1}{2}c\tilde{a}^2$. Substituting the contract and taking the expectation over both the exogenous shock and mistakes, the agent's expected utility is

(4.1) $\qquad U = -\exp\{-\rho[f + sa - \frac{1}{2}ca^2 - \frac{1}{2}\rho\sigma^2_\varepsilon s^2]\}$

$$\times[\exp\{-\frac{1}{2}\rho^2\sigma^2_\upsilon\beta(s - ca)^2\}\,[\beta(2/\sigma^2_\upsilon)N(0; \rho\sigma^2_\upsilon\beta(s - ca)^2, \sigma^2_\upsilon)],$$

where $\beta = 1/\sqrt{(\rho c\sigma^2_\upsilon - 1)}$. The first term in the product is the familiar expression for agent's expected utility when the agent does not make mistakes. The second two terms are the effect of negative mistakes on the agent's expected utility. The second term is the effect of the increased variation in compensation due to mistakes while the third term is the risk-adjusted truncated distribution. Note that the agent's intended effort enters both of these terms through the expression $(s - ca)^2$.

The agent's expected utility maximizing intended effort is once again $a = s/c$. Substituting the agent's intended effort into the expression for expected utility, the last two terms reduce to β/σ^2_υ. The agent's participation constraint then simplifies to (2.2). The principal's expected net profit is

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[5] The distribution of mistakes is $F(\upsilon; 0, \sigma^2_\upsilon) = 2\Phi(\upsilon^2/\sigma^2_\upsilon; 0, \sigma^2_\upsilon)$, for $\upsilon \leq 0$, where Φ is the standard normal distribution.

(4.2) $\quad s/c - f - s^2/c - (1-s)\sigma^2_\upsilon\sqrt{2/\pi};$

the last term is the principal's share of the expected value of mistakes. As a result, the agent's share of output is

(4.3) $\quad s = (1 + c\sigma^2_\upsilon\sqrt{2/\pi})/[1 + \rho c\sigma^2_\varepsilon].$

The term $c\sigma^2_\upsilon\sqrt{2/\pi}$ in the numerator adjusts the agent's share upward to make the agent bear part

of the lower expected output due to negative mistakes. The increased variability due to mistakes

is hedged and does not affect the agent's share of output.

B. Costless Mistakes. I assume the cost of effort depends on intended effort, $\gamma(a) = \frac{1}{2}ca^2$.

Substituting the contract and taking the expectation over both the exogenous shock and mistakes,

the agent's expected utility is

(4.4) $\quad U = -\exp\{-\rho[f + sa - \frac{1}{2}ca^2 - \frac{1}{2}\rho\sigma^2_\varepsilon s^2]\}$

$$\times [\exp\{-\frac{1}{2}\rho^2\sigma^2_\upsilon s^2\} [(2/\sigma^2_\upsilon)N(0; -\rho\sigma^2_\upsilon s, \sigma^2_\upsilon)],$$

This expression has the same interpretation as (4.1); the second and third terms are the effects of

increased variability and the risk-adjusted truncated distribution. The agent chooses effort to

maximize expected utility, given the contract. This again yields $a = s/c$.

The principal's expected net profit is again given by (4.2). The agent's participation

constraint becomes

(4.5) $\quad -\exp\{-\rho[f + \frac{1}{2}s^2/c - \frac{1}{2}(\sigma^2_\varepsilon + \sigma^2_\upsilon)s^2]\}\{(2/\sigma^2_\upsilon)N(0; -\rho\sigma^2_\upsilon s, \sigma^2_\upsilon)\} = u(\overline{w}).$

Solving for the contract, the agent's share of output satisfies

(4.6) $\quad s = [1 + c\sigma^2_\upsilon\sqrt{2/\pi} + c\sigma^2_\upsilon\eta(s)]/[1 + \rho c(\sigma^2_\varepsilon + \sigma^2_\upsilon)],$

where $\eta(s) = (\partial N(0; -\rho s\sigma^2_\upsilon, \sigma^2_\upsilon)/\partial\mu)/N(0; -\rho s\sigma^2_\upsilon, \sigma^2_\upsilon)$. Again, the agent's share of output is

adjusted upward by $c\sigma^2_\upsilon\sqrt{2/\pi}$ to compensate for the negative expected value of mistakes. The

term $c\sigma^2_\upsilon\eta(s)$ in the numerator is the marginal effect of the risk-adjusted truncated distribution and is negative. This term and the term $\rho c\sigma^2_\upsilon$ in the denominator adjust the agent's share of output downward to adjust for the agent's inability to hedge the risk and reduce the risk born by the agent.

When mistakes are always negative, they have the direct effect of reducing output by the expected value of mistakes, $\sigma^2_\upsilon\sqrt{(2/\pi)}$, in addition to any indirect effects through the contract. The agent's mistakes are costly to the principal and there is value to the principal of training that reduces the variance of mistakes and increases expected output. The effect of training on incentives is ambiguous when mistakes do not affect the cost of effort. When mistakes affect the cost of effort, training leads to weaker incentives for the agent.

When mistakes are always negative, the agent's share of output may be greater than one. If the agent's share is greater than one then there are two possibilities, depending on the parameter values. One possibility is that the agent's fixed compensation or salary is negative, that is, the agent pays the principal. The other possibility is that the principal's participation constraint is violated, that is, the principal may prefer not contracting to contracting with an agent who is too prone to mistakes.

5. Conclusion.

The purpose of this paper is to examine the effects of agent's mistakes on the outcomes of the principal-agent model, using the linear-exponential-normal model. I make two distinctions. I distinguish between the cases where mistakes may be both positive and negative and the case where mistakes are always negative. I also distinguish between the cases where the cost of effort depends on intended effort or actual effort. When mistakes may be positive or negative, there is

no change in expected output, but when mistakes are always negative, expected output is reduced. Mistakes also increase the variance of output. When the cost of effort depends on actual effort, the increased variability in effort cost creates a natural hedge against the increased variability in the agent's compensation. When the cost of effort depends on intended effort, this natural hedge is absent.

Thus, when mistakes may be positive or negative and the cost of effort depends on actual effort, the optimal linear contract is the same as when the agent never makes mistakes. When mistakes are always negative and the cost of effort depends on actual effort, the agent's share of output is adjusted upward since the agent must bear part of the expected decrease in output. When mistakes may be positive or negative and the cost of effort depends on intended effort, the agent's share of output is adjusted downward to mitigate the increase in the variability of output. When mistakes are always negative and the cost of effort depends on intended effort, the agent's share of output is adjusted upward to reflect the decrease in expected output, and adjusted downward to mitigate the increased variability of output. The agent's share of output may be larger or smaller then when the agent never makes mistakes.

Monitoring may decrease the variance of the exogenous shock, yield more precise information on the agent's effort and increase the agent's share of output. This is true whether or not the agent makes mistakes, and implies there is a positive correlation between monitoring and incentives. Monitoring is distinguished from training, which decreases the variance of mistakes. Depending on the circumstances, the effect of training on the agent's share of output may be positive, negative or zero which implies there is not necessarily a correlation between training and incentives.

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